ESTIMATING THE DIMENSIONS OF THE RADIAL CRACK ZONE FORMED IN A CONTAINED EXPLOSION OF A LINE CHARGE IN A BRITTLE MEDIUM

P. A. Martynyuk and E. N. Sher UDC 622.235

A scheme has been proposed [1, 2] for the growth of a radial crack system formed by explosion of a line charge in a brittle medium, and a method has been given for calculating the dynamics of such cracks. In approximate calculations in [2], a quasistatic solution was used for the state of strain in an elastic plane having a radial system of lines of section loaded from within a given load.

In a confined explosion, the loading pressure from the detonation products in the cavity is not known in advance and must be found at the same time as the motion of the medium is determined and the growth of the radial cracks is established.

Here we estimate the maximum dimensions of the cracks arising in a confined explosion from a line charge in a brittle medium. We envisage cases where the detonation products do or do not penetrate into the cracks. Allowance is also made for the rock pressure. A comparison is made of the theory with experiments on explosions in lucite.

1. Quasistatic Zone Model for the Confined Explosion of a Line Charge in a Brittle Medium. The quasistatic approximation has to be used [3] because there are difficulties in solving dynamic problems in the theory of elasticity involving growing cracks.

We consider the final stage of a confined explosion for a line charge in a brittle medium. In the explosion cavity of radius r_0, there is a certain pressure p due to the detonation products. There is a zone of plastic strain $r_0 < r < r_1$ around the cavity, where the material is in the plastic or crushed (powder) state as a result of the pressure. Outside the plastic zone at $r_1 < r < l$ lies a system of radial cracks, the longest of which are in critical equilibrium.

The rock pressure far from the explosion cavity is given. One has to calculate the equilibrium state of this configuration.

This formulation differs from an earlier one [3] in that there is no zone of columnar elasticity. Here the zone of radial cracks together with the remaining elastic space may be considered together as a whole. In that approach, the condition for damage in terms of the limiting tensile stress for the columnar elasticity zone is replaced by a condition for limiting equilibrium in the cracks, which is sounder for a brittle medium. An adiabatic pressure-variation law is assumed for the gases in the cavity.

In the calculations, we use the Jones-Miller adiabat [4] for a cylindrical trotyl charge in the form

$$p(r_0)=\begin{cases} p_0\left(\dfrac{r_0}{r_{00}}\right)^{-2\gamma_1}, & r_0<r_0^*,\\ p_0\left(\dfrac{r_0^*}{r_{00}}\right)^{-2\gamma_1}\left(\dfrac{r_0}{r_0^*}\right)^{-2\gamma_2}, & r_0>r_0^*,\end{cases} \quad (1.1)$$

$$p_0=10^{10}\ \text{Pa},\quad \gamma_1=3,\quad \gamma_2=1.27,\quad r_0^*/r_{00}=1.89,$$

where r_{00} is the initial charge radius and r_0 is the cavity radius.

To describe the state of stress in the plastic zone, we follow [4] in taking the Coulomb law $\tau = C - \sigma \tan\varphi$, where τ and σ are the tangential and normal stresses on a shear area. In terms of the principal stresses for the axially symmetrical case, we have

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 127-132, July-August, 1984. Original article submitted June 1, 1983.

0021-8944/84/2504-0619$08.50

$$(1+\beta)\sigma_\theta - \sigma_r - Y = 0,$$
$$Y = 2C\cos\varphi/(1-\sin\varphi),\ \beta = 2\sin\varphi/(1-\sin\varphi). \tag{1.2}$$

We use the equation of equilibrium

$$d\sigma_r/dr - (\sigma_r - \sigma_\theta)/r = 0$$

and (1.2) to get for the plastic zone that

$$\sigma_r = Y/\beta + B/r^{\beta/(1+\beta)},$$

where B is an arbitrary constant. At the outer boundary r_1 of the plastic zone, the condition applies for damage in the medium under uniaxial compression $\sigma_r|_{r_1} = -\sigma_c$, where σ_c is the strength in compression. Then we have for the distribution of σ_r that

$$\sigma_r = Y/\beta - (Y/\beta - \sigma_c)(r_1/r)^{\beta/(1+\beta)}. \tag{1.3}$$

The material in the plastic zone is assumed to be incompressible:

$$r_0^2 - r_{00}^2 = r_1^2 - (r_1 - u_1)^2 \tag{1.4}$$

(u_1 is the radial displacement of points in the medium at the boundary of the plastic zone). Here we neglect dilatancy effects in order to simplify the problem.

The system of radial cracks begins at the circular boundary $r = r_1$. The radial stress at this boundary is $-\sigma_c$. We calculate the equilibrium with the following assumptions: a) there are n cracks of maximal size, b) the state of stress in these cracks is independent of the presence of the other shorter ones, and c) to calculate the displacement u_1 of the boundary one can use equations for the columnar elastic zone, which stimulates the radial-crack system.

The data of [5] can be used to describe the state of stress at the vertices of the n longest cracks. Here the calculations are based on the following approximate relationship for the intensity coefficient. For cracks extending only slightly from the boundary of the plastic zone we have [6]

$$K_I \approx 1.12\sigma_c\sqrt{\pi(l - r_1)}, \tag{1.5}$$

where l is the radius at which the crack vertex lies.

For long cracks we have [7, 8]

$$K_I \approx \sigma_c r_1\sqrt{\pi}/\sqrt{nl} - 2P\sqrt{\pi l}/\sqrt{n}. \tag{1.6}$$

We assume that the intermediate case is described by (1.5) and (1.6), which give a smaller value. The second term in (1.6) incorporates the external rock pressure P.

To provide a linkup at the outer boundary of the plastic zone, one has to estimate the mean displacement of the circular boundary to the system of radial cracks. It follows from assumption c above that the mean radial displacement at radius l is

$$u_l = \frac{1+\nu}{E}\, l\left[\frac{\sigma_c r_1}{l} - 2P(1-\nu)\right].$$

The assumption of incompressibility in the crack zone then gives

$$u_1^{\mathrm{i}} = \left(\frac{l}{r_1}\right)u_l.$$

The assumption about the columnar zone gives

$$u_1^{\mathrm{c}} = u_l + \sigma_c r_1(1-\nu^2)\ln(l/r_1).$$

We take

$$u_1 = \min\left(u_1^{\mathrm{i}}, u_1^{\mathrm{c}}\right), \tag{1.7}$$

to complete the description of the radial-crack zone.

By solving (1.1) with (1.3) on the basis that $r = r_0$ and (1.4)-(1.7) and using $K_I = K_{Ic}$, we can find the equilibrium values of r_0, r_1, and l with given values for the parameters of the medium E, σ_c, K_{Ic}, ν, C, φ, those of the charge r_{00}, p_0, γ_1, γ_2, r_0^*/r_{00}, and the rock pressure P.

We took the following values for lucite: $E = 3\cdot10^9$ Pa, $\sigma_c = 10^8$ Pa, $K_{Ic} = 10^6$ Pa$\cdot$m$^{1/2}$, $\nu = 0.3$, $C = 0$, and $\varphi = 30°$.

Curves 1-3 of Fig. 1 show the results of charges of initial radii r_{00} = 1, 2, and 3 mm correspondingly. It is evident from Fig. 1 that the sizes of the radial cracks decrease monotonically as the rock pressure rises. The steepest fall occurs in the range of pressures comparatively low for mine conditions (up to $5 \cdot 10^6$ Pa). This may explain the considerable increase in the specific consumption in drilling explosions even at small depths and when there are free surfaces.

Another feature of Fig. 1 is the deviation from geometrical similarity in the dimensions of the radial cracks as the charge radius varies. For example, the ratio of the crack radius to the charge radius increases as the charge radius increases.

This deviation is the most pronounced in the absence of rock pressure, and it is largely inappreciable at pressures over $5 \cdot 10^9$ Pa. Geometrical similarity applies closely for the calculated size of the plasticity zone.

2. Effects of Gas Penetration into Cracks on the Final Size. It has been shown [9, 10] that the penetration of detonation products into the cracks may be appreciable during the damage process. This factor also has a substantial effect on the final crack size.

We consider the following model for the final stage in crack growth produced by an explosive charge. In a plane there is a hole of radius r_0 with radial cracks of length l emerging from it. The explosion products propagate in the cracks and exert a pressure p on the sides of the cracks and in the charge cavity. The temperature of the explosion products is close to that of the medium. The gas pressure p can be determined from the volume of the cavity and cracks by means of the gas law if one knows the volume of the gases under normal conditions. The external pressure at infinity is P.

The end to the damage process means that the cracks are in limiting equilibrium and that the stress-intensity coefficient at the vertices is equal to the critical value for crack halt. This additional condition enables one to find for example the crack length if the other parameters are known.

We now give an approximate solution. We first derive the gas pressure p. For this purpose we estimate the volume of the cavity and cracks. We assume that within a circle of radius l passing through the vertices of the cracks there is a radial stress of magnitude p. Then the medium within the circle $r < l$ is compressed by the hydrostatic pressure p. Then in the case of two-dimensional deformation, the change in the volume of the rock within the circle $r < l$ is

$$\Delta v_1/v_1 = (2p/E)(1-2\nu)(1+\nu), \quad v_1 = \pi\left(l^2 - r_0^2\right).$$

The displacement of the circle of radius l by the internal pressure p and the external pressure P leads to an additional change in volume within the circle $r < l$

$$\Delta v_2 = -\pi l^2 2(1-2\nu)(1+\nu)P/E + 2\pi(p-P)l^2(1+\nu)/E.$$

The total increment in the gas volume due to cavity expansion and crack opening for $l \gtrsim 3r_0$ is estimated by

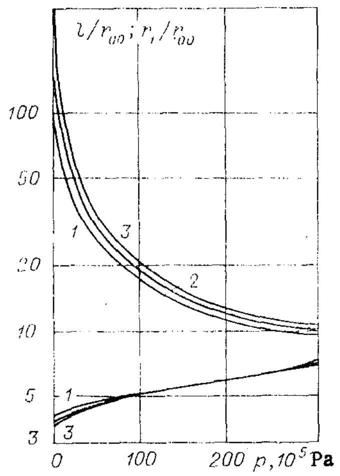

Fig. 1

$$\Delta v = [4\pi l^2(1 - \nu^2)/E](p - P).$$

Then if the volume of gas per unit charge length is V_0 under normal conditions ($p = p_n$, $T = T_n$), we have

$$p\left(\pi r_0^2 + \Delta v\right) = p_n V_0.$$

We solve this equation for p to get

$$\bar{p} = \left[A\bar{P}\bar{l}^2 - 1 + \sqrt{1 + A^2\bar{P}^2\bar{l}^4 + 2A\left(2\bar{p}_0 - \bar{P}\right)\bar{l}^2}\right] \Big/ (2A\bar{l}^2). \tag{2.1}$$

Here

$$\bar{p} = p/E,\ \bar{P} = P/E,\ \bar{p}_0 = p_n V_0/\left(\pi E r_0^2\right),\ \bar{l} = l/r_0,\ A = 4(1 - \nu^2).$$

To find the intensity coefficient at the vertices of the cracks loaded from within by the pressure p of (2.1) in the presence of the external pressure P, we use the solution to Westman's problem [8] on the state of stress in a star of n cracks equal in length distributed at identical angles and subject to internal pressure.

For our treatment, we neglect the size of the cavity by comparison with the crack length $l - r_0$ to get for the case of limiting equilibrium that

$$K_I \approx (2\sqrt{\pi l/n})(p - P) = K_{Ic}$$

or in dimensionless form

$$\bar{K}_{Ic} = 2\sqrt{\frac{\pi\bar{l}}{n}}(\bar{p} - \bar{P}) = \sqrt{\frac{\pi}{n}}\left(\frac{-1 - A\bar{P}\bar{l}^2 + \sqrt{1 + A^2\bar{P}^2\bar{l}^4 + 2A\left(2\bar{p}_0 - \bar{P}\right)\bar{l}^2}}{A\bar{l}^{3/2}}\right), \tag{2.2}$$

where $\bar{K}_{Ic} = K_{Ic}/(E\sqrt{r_0})$; formula (2.2) relates the parameters of the charge (V_0, r_0) and of the medium (P, K_{Ic}, E, ν) to the crack length l and number of cracks n.

Formula (2.2) simplifies for P = 0:

$$\bar{K}_{Ic} = \sqrt{\frac{\pi}{n\bar{l}}}\left(\frac{-1 + \sqrt{1 + 4A\bar{p}_0\bar{l}^2}}{A\bar{l}}\right). \tag{2.3}$$

For $\overline{4A p_0 l^2} \gg 1$ or for example for $\bar{p}_0 \geqslant 10^{-2}$, $\bar{l} > 20$

$$\bar{K}_{Ic} \simeq \sqrt{\frac{\pi}{n\bar{l}}}\frac{2\sqrt{\bar{p}_0}}{\sqrt{A}},\quad l \simeq \frac{E p_n V_0}{n K_{Ic}^2(1 - \nu^2)}. \tag{2.4}$$

Figure 2 shows the results from (2.3) with n = 5 and ν = 0.3 in the form of isolines for l/r_0 in the plane of $\bar{K}_{Ic} = K_{Ic}/(E\sqrt{r_0})$, $\bar{p}_0 = p_n V_0/(\pi E r_0^2)$ on a logarithmic scale.

One can trace the effects of P on the size of the damage zone from Fig. 3, which shows graphs constructed from (2.2) for various P relating $\bar{K}_{Ic}$ to l/r_0 for $\bar{p}_0 = 10^{-2}$.

These calculations enable one to estimate the maximum crack size for various charges and for a wide range of rock parameters for the case where the explosion is performed under conditions homogeneous with respect to angle and the number of cracks of maximum length is close to 5.

The approximate limits to the parameters are as follows. Young's modulus E for rocks varies over the range 10^9–10^{11} Pa, while Poisson's ratio is ν = 0.2–0.4. At present we have little data on K_{Ic} for rocks. We give here the results of [11] according to which K_{Ic} = 3 MPa·m$^{1/2}$ for sandstone and marble. The rock pressure in most cases varies from 0 up to $5 \cdot 10^7$ Pa. The gas production from commercial explosives is [12] 100–700 liter/kg under normal conditions.

As an example, we calculate the dimensions of the cracks from the explosion of a charge of PZhV-20 explosive with a density of 1 g/cm³ placed in a bored hole of diameter 80 mm in a sandstone under conditions of rock pressure $P = 1.5 \cdot 10^7$ Pa. We find the values of the dimensionless parameters for

$$E = 3 \cdot 10^{10}\ \text{Pa},\quad p_n V_0/\left(\pi r_0^2\right) = 3 \cdot 10^7\ \text{Pa},$$

$$\bar{P} = P/E = 0.5 \cdot 10^{-3},\ \bar{K}_{Ic} = K_{Ic}/(E\sqrt{r_0}) = 5 \cdot 10^{-4},$$

$$\bar{p}_0 = p_n V_0/\left(\pi E r_0^2\right) = 10^{-3}.$$

From (2.2) we get $\bar{l} = 155$, $l = 6.2$ m.

To evaluate the likeliness of the results, we compare them with some test data. In practice, one can disrupt a sandstone by exploding a series of charges of diameter 40 mm involving a consumption of explosive of 40 g/m (4 turns of DSh with sand filling). The distances between the charges should be 0.5 m. In that case, $\bar{p}_0 = 5 \cdot 10^{-5}$, $\bar{K}_{Ic} = 7 \cdot 10^{-4}$, and from (2.2) we have $l = 160 r_0 = 3.2$ m. It is evident that the calculated crack lengths are much greater than the distances between charges used in practice, because the theory gives an overestimate of crack sizes, since it is assumed that all the detonation products operate in the final phase of damage. In fact, some of the gas may penetrate into the rock during the damage and be absorbed or may simply escape. These factors have a marked effect on the final size of the cracks. The above theoretical estimates could be revised by incorporating these factors, if they occur.

To elucidate the effects of gas penetration on the size, we used the nomograms of Fig. 2 to determine the crack lengths of a trotyl charge in Lucite for the parameters used in Fig. 1 without penetration. The penetration scheme gave crack lengths about two times larger than those without penetration.

3. Measurement of Radial-Crack Zone Parameters Due to the Explosion of a Long Charge in Lucite. The explosions were performed in sheet Lucite of thicknesses 4.2-220 mm. The explosive was TEN. The charge was placed in a hole drilled in the sheet perpendicular to the plane.

Figure 4 shows the typical disposition of the charge in a sheet for the thickness ⩾ 70 mm; the charge was detonated electrically using lead azide. The plug was produced with epoxide resin placed in a channel bearing an M5 thread.

The cracks were divided by length into groups: the longest ones, shorter ones, and so on. For each group we derived the mean crack size and the number of cracks of that length or range.

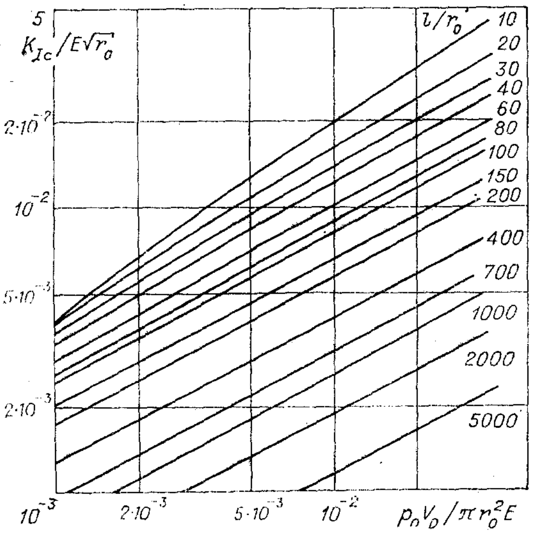

Fig. 2

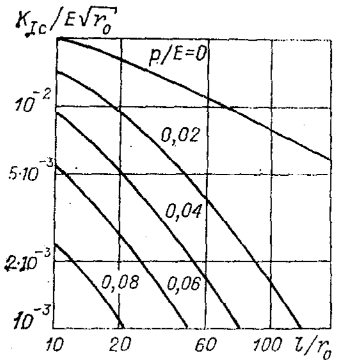

Fig. 3

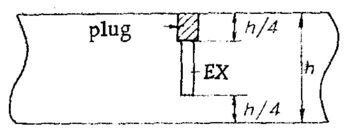

Fig. 4

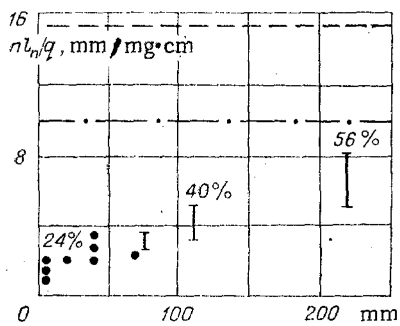

Fig. 5

TABLE 1

Experiment No.	Amount q, mg/cm	Specimen thickness, mm	Plug	Charge diameter, mm	Number of cracks n	Crack radius l_n, mm	nl_n	$\frac{nl_n}{q}$
1	75	77	—	2,4	9—11	25	225—275	3—3,6
2	96,8	110	—	2,4	12—14 3	27 100	324—378 300	3,3—3,9
3	75	110	epoxide	2,4	10—12 4	30 70—90	300—360 280—360	4—4,8
4	102	110	—	2,8	8—9	60	480—540	4,7—5,3
5	108	19	—	2,8	16 8	15 25	240 200	2,2
6	67	220	epoxide	2,4	8—10 4	60 100	480—600 400	6—7,2
7	54	4,2	Lucite	2	7	15	105	2

Table 1 collects the data, including the product of the number of cracks and the length as determined from the above nl_n. According to (2.4), this product is a constant for each explosion.

It follows from (2.4) that if one uses a single material and explosive, the product of the crack length by the number of cracks should be constant when this quantity is divided by the amount of explosive per unit length (nl_n/q); this quantity is also given in the table. The results show that nl_n is constant for each explosion. Figure 5 gives data on nl_n/q, which show that nl_n/q increases monotonically with the lucite thickness. One naturally expects that this effect will become less as the sheet thickness increases. To compare the data with the above theory, we calculated the failure parameters for a TEN charge of diameter 2.4 mm in Lucite. For this material, $E = 3 \cdot 10^9$ Pa and $K_{Ic} = 1 \cdot 10^6$ Pa·m$^{1/2}$. The gas release from TEN is 500 liters/kg.

The consumption of explosive was 0.067 g/cm. The data imply that $\bar{K}_{Ic} = 10^{-2}$, $\bar{p}_0 = 0.025$, and if the gases penetrate into the cracks, then $l/r_0 = 180$ according to Fig. 2, so the calculated value is $nl_n/q = 16$.

This value is represented by the dashed line in Fig. 5. For certain explosions we calculated the amount of gas necessary to produce the damage actually found. The data are given in Fig. 5 as the percentage of the amount of gas actually released, and the values are quite likely.

Another estimate can be made of the size of the radial-crack zone by reference to the model of section 1, in which it was assumed that the detonation products are retained in the cavity and do not enter the cracks. With this assumption, Fig. 1 shows that a charge of diameter 2.4 mm in Lucite gives five cracks of size $115r_0$ = 138 mm, and then $nl_n/q = 10$, which is shown as the dot-dash line in Fig. 5. It is evident that the experimental data approach this level quite closely.

One expects that the experimental data will approach the upper theoretical asymptote as the plate thickness increases further. This confirms the theoretical estimates of Sections 1 and 2.

LITERATURE CITED

1. E. N. Sher. "Estimating the crushing action of a long charge in a brittle medium," FTPRPI, No. 1 (1975).
2. E. N. Sher, "An example of calculating the motion of radial cracks formed by explosion in a brittle medium from the quasistatic approximation," FTPRPI, No. 2 (1982).
3. V. N. Rodionov, A. N. Romashev, et al., The Mechanical Effects of an Underground Explosion [in Russian], Nedra, Moscow (1971).
4. P. Chadwick, A. Cox, and H. Hopkinson, Mechanics of Deep Underground Explosions [Russian translation], Mir, Moscow (1966).
5. F. Ouchterlony, "Fracture mechanics applied to rock blasting," Rept/Swedish Detonic Res. Found.: DS 1973:29, Stockholm (1973).

6. G. P. Cherepanov, Mechanics of Brittle Failure [in Russian], Nauka, Moscow (1974).
7. E. N. Sher, "A case of equilibrium in a radial crack system," Zh. Prikl. Mekh. Tekh. Fiz., No. 5 (1974).
8. R. A. Westman, "Pressurized star crack," J. Math. Phys., 43, No. 3 (1964).
9. V. M. Kuenetsov and A. F. Shatsukevich, "The interaction of detonation products with the laws of the explosion cavity in soils and rocks," Fiz. Goreniya Vzryva, 13, No. 5 (1977).
10. V. M. Komir, V. M. Kuznetsov, and A. F. Shatsukevich, "The effects of gaseous detonation products on the damage to rocks from explosion," FTPRPI, No. 6 (1979).
11. A. P. Bobryakov, G. N. Pokrovskii, and B. N. Serpeninov, "A study of the damage from impact in flat specimens," in: Aspects of Rock Damage Mechanisms [in Russian], IGD Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1976).
12. D. V. Dubnov, N. S. Bakharevich, and A. I. Romanov, Commercial Explosives [in Russian], Nedra, Moscow (1973).

NUMERICAL MODELING OF THE PROCESS OF PENETRATION OF A RIGID BODY OF REVOLUTION INTO AN ELASTOPLASTIC BARRIER

V. I. Kondaurov, I. B. Petrov, and A. S. Kholodov

UDC 539.3

We will consider the axisymmetric problem of the penetration of an absolutely rigid body of revolution into a deformable barrier of finite thickness. The rheology of the barrier material is described by the equations of flow of elastoplastic bodies. Important aspects of these problems are the sharply expressed wave character of the solution and the large deformations suffered by the barrier. Penetration problems have been the subject of a large number of experimental investigations, which have been used as a basis for studying the effect of various controlling parameters and observable effects and constructing various approximate methods of calculating penetration processes. However, a fully detailed picture of the processes of interaction of projectiles and deformable targets can be obtained only by means of the numerical solution of problems of this kind on the basis of various rheological models and a subsequent comparison with the experimental results in order to refine the mathematical model.

The complex nature of these problems imposes rigid constraints on the choice of a numerical method of solution, the choice of independent variables, etc. In particular, for large penetration depths the use of traditional Lagrangian variables leads to considerable distortions (and often to a loss of regularity) of the difference net and the need to reconstruct it periodically (which may lead to a significant loss of accuracy). The use of fixed Eulerian coordinates leads to difficulties in formulating the boundary conditions at the surface of the barrier and the need to choose a difference net with a large number of nodes in order to obtain acceptable accuracy in a continuous calculation without explicit isolation of the barrier surface. Both these approaches have been used in the numerical solution of problems of this kind, for example, in [1-5]. Here we shall use a moving coordinate system (tied to the upper and lower edges of the barrier) and the net-characteristic method [6], which allows the most natural construction of the computational algorithm near the edges of the region of integration and to a certain extent makes it possible to take into account the region of variation and the wave character of the solution. This explicit scheme of the first order of accuracy is one of those with a positive approximation (monotonic and majorant schemes, to use another terminology) and, as shown in [7], has minimum approximation viscosity among the explicit two-layer schemes of this kind, which is an important property in the continuous calculation of nonsmooth (discontinuous) solutions without explicit isolation of the surfaces of discontinuity [8].

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 132-139, July-August, 1984. Original article submitted June 8, 1983.

0021-8944/84/2504-0625$08.50